Precalculus Learning Goals - Week 6

This week we'll talk about **Rational and Algebraic Functions** in detail, and about **Modeling with Functions**.

The general goals for the section **Rational and Algebraic Functions** are as follows. At the end of this section, students should be able to:

- *Understand how asymptotes and holes can arise with rational and algebraic functions.*

More specifically, at the end of this week you should be able to:

- Find the holes of a rational function through factoring.
- Find the end behavior of a rational function.
- Find the vertical asymptotes of a rational function.
- Use "as $x \to a^{\pm}$, $f(x) \to b$" notation appropriately, for $a, b \in (-\infty, \infty)$.
- Describe vertical asymptotes of an algebraic function.
- Describe the end behavior of an algebraic function.

Sample Problems. Here are some sample problems, of the type that you would do to demonstrate that you've learned the material. These are not the only types of problems you may see – they're just a sample.

- Find the holes, end behavior, and vertical asymptotes of $g(x) = \frac{2x^2-2x-4}{x^2-1}$. As $x \to 1^-, f(x) \to$??. As $x \to -1^+, f(x) \to$??.
- Let $f(x) = \frac{3}{1+\sqrt{x}}$. Then as $x \to -\infty, f(x) \to$??. What is the domain of f? Does f have any vertical asymptotes?
- True or False: if a rational function is given as $f(x) = \frac{g(x)}{h(x)}$, and if $h(2) = 0$, then f must have a vertical asymptote at 2. Does the answer change if you know that $g(2) = 5$?

The general goals for the section **Modeling with Functions** are as follows. At the end of this section, students should be able to:

- *Create functions that model real-world situations.*

More specifically, at the end of this week you should be able to:

- Write a formula for a function that models a real-world situation.
- Interpret the graph of a function that models a real-world situation.
- Sketch a rough graph of a function corresponding to a real-world situation.

Sample Problems. Here are some sample problems, of the type that you would do to demonstrate that you've learned the material. These are not the only types of problems you may see – they're just a sample.

- T-shirts Unlimited will print custom t-shirts for you. The company charges \$40 to set up for printing, and then \$10 per shirt, unless you buy more than 20 shirts, in which case the price drops to \$8 per shirt and the set-up fee is waived. Write a formula for the function $C(x)$ that represents the cost to print x shirts.
- Write a formula for a function that expresses the area of a circle as a function of its circumference.

- The Richter scale for earthquakes defines the magnitude of an earthquake to be $M = \log\left(\frac{I}{S}\right)$, where I is the intensity of the earthquake and S is the intensity of a "standard" earthquake, which is one whose amplitude is 1 micron when measured on a seismograph located 100 miles from the epicenter of the earthquake. What is the magnitude of a standard earthquake? The 1989 San Francisco earthquake measured 6.9 on the Richter scale. The 1906 San Francisco earthquake, however, measured 8.3. How many times more intense was the 1906 earthquake?
- Below is the graph of the distance traveled by two runners in a race (graph omitted). Describe in words how each runner ran the race. When was Runner A running fastest? Who won?
- Jack is driving from City A to City C, traveling through City B. City A is 40 miles from City B, and City C is 60 miles from City B. If Jack drives at a constant rate of 20 mph (he's a safe driver), sketch a graph of Jack's distance from City B. Can you write a formula to describe the function that represents Jack's distance from City B?